SOLUTIONS

Joint Entrance Exam | IITJEE-2018

Paper Code - B

8th April 2017 | 9.30 AM – 12.30 PM

PART-B	MATHEMATICS

31.(2) $x = \frac{y+7}{2} - 6 \quad \Rightarrow \quad 2x = y + 7 - 12 \quad \Rightarrow \quad 2x = y - 5 \quad \Rightarrow \quad 2x - y + 5 = 0$

Also, centre of the circle is $(-8, -6)$ and the radius is $\sqrt{64 + 36 - c}$

$\Rightarrow \quad \left(\frac{-16+6+5}{\sqrt{5}}\right) = \sqrt{100 - c} \quad \Rightarrow \quad \sqrt{5} = \sqrt{100 - c} \quad \Rightarrow \quad c = 95$

32.(4) $(2+\lambda)x - (2+\lambda)y + (3+\lambda)z - 2 + \lambda = 0$

$(1+3\mu)x + (2-\mu)y + (2\mu - 1)z - 3 - \mu = 0$

$\Rightarrow \quad \frac{2+\lambda}{1+3\mu} = \frac{-(2+\lambda)}{2-\mu} \quad \Rightarrow \quad \mu - 2 = 1 + 3\mu \quad \Rightarrow \quad 2\mu = -3 \quad \Rightarrow \quad \mu = \frac{-3}{2}$

So the equation of plane is $7x - 7y + 8z + 3 = 0$

Now, distance from origin equal to $\left|\frac{3}{\sqrt{7^2 + 7^2 + 8^2}}\right| = \frac{1}{3\sqrt{2}}$

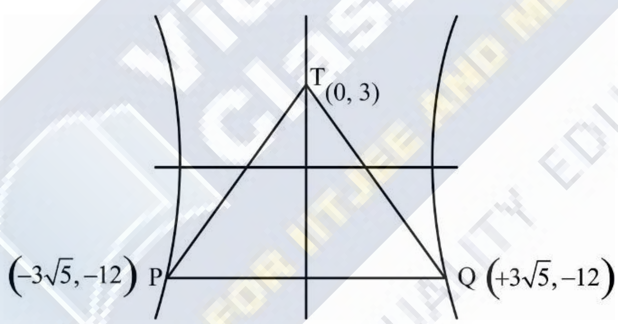
33.(1) $x^2 - x + 1 = 0 \quad \Rightarrow \quad x = \frac{1 \pm \sqrt{-3}}{2} = -\omega, -\omega^2$ (where ω and ω^2 are non-real cube roots of unity)

$\Rightarrow \quad \alpha = -\omega$ and $\beta = -\omega^2$

$\Rightarrow \quad (-\omega)^{101} + \left(-\omega^2\right)^{107} = -\left(\omega^{101} + \omega^{214}\right) = -\left(\omega^2 + \omega\right) = 1$

34.(3) Equation of PQ,

$4x \cdot (0) - 3y = 36$



$y = -12$

Area of $\Delta TPQ = \frac{1}{2} \times 15 \times 6\sqrt{5} = 45\sqrt{5}$

35.(2) $2yy' = 6$

$y' = \frac{6}{2y} = \frac{3}{y_1}$

$18x_1 + 2by_1 y' = 0$

$y' = -\frac{18x_1}{2by_1} = \frac{-9x_1}{by_1} \quad \Rightarrow \quad -\frac{27x_1}{by_1^2} = -1 \quad \Rightarrow \quad b = \frac{27x_1}{y_1^2}$

$y_1^2 = 6x_1 \quad \Rightarrow \quad b = \frac{9}{2}$

36.(4) $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$\Rightarrow \quad k = \frac{7}{2}$

$x + ky + 3z = 0$ …..(i)

$3x + ky - 2z = 0$ ….(ii)

$2x + 4y - 3z = 0$ …..(iii)

On solving (i) and (ii)

$2x - 5z = 0$ …..(iv)

On solving (iii) and (iv)

$4y = -2z$

$$\frac{xz}{y^2} = \frac{\frac{5}{2} z \times z}{\frac{z^2}{4}} = 10$$

37.(1) $2\left|\sqrt{x} - 3\right| + \sqrt{x}\left(\sqrt{x} - 6\right) + 6 = 0$

Case-I: $\sqrt{x} \geq 3$

$\Rightarrow \quad 2\left(\sqrt{x} - 3\right) + x - 6\sqrt{x} + 6 = 0 \quad \Rightarrow \quad x - 4\sqrt{x} = 0 \quad \Rightarrow \quad x = 0, 16$

As $x \geq 9 \quad \Rightarrow \quad x = 16$

Case-II: $\sqrt{x} < 3 \quad \Rightarrow \quad -2\sqrt{x} + 6 + x - 6\sqrt{x} + 6 = 0 \quad \Rightarrow \quad x - 8\sqrt{x} + 12 = 0$

$\Rightarrow \quad \left(\sqrt{x} - 6\right)\left(\sqrt{x} - 2\right) = 0 \quad \Rightarrow \quad x = 36, 4$

As, $\sqrt{x} < 3 \quad \Rightarrow \quad x = 4$

$\therefore$ There are exactly two elements in the given set.

38.(4) $8\cos x \cdot \left[\left(\cos^2 \frac{\pi}{6} - \sin^2 x\right) - \frac{1}{2}\right] = 1$

$8\cos x\left(\frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x\right) = 1$

$\frac{8\cos x}{4} \times \left(4\cos^2 x - 1 - 2\right) = 1$

$\cos 3x = 4\cos^3 x - 3\cos x$

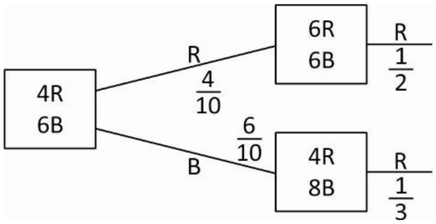
$2 \times \cos 3x = 1$

$\cos 3x = \frac{1}{2}$

$3x \in [0, 3\pi]$

$3x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3} \quad \Rightarrow \quad \text{Sum} = \frac{13\pi}{9}.$

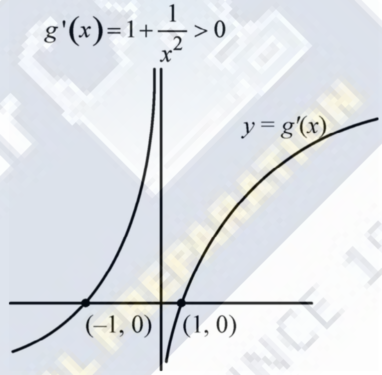
39.(4)



Total probability $= \frac{4}{10}\cdot\frac{1}{2}+\frac{6}{10}\cdot\frac{1}{3}=\frac{2}{5}$

40.(2) Let $g(x) = x - \frac{1}{x} = t$

$g'(x) = 1 + \frac{1}{x^2} > 0$



$\therefore \quad t \in R - \{0\};\ t^2 \in (0, \infty)$

$\therefore \quad f(x) = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 = t^2 + 2 \in (2, \infty)$

$\therefore \quad h(x) = \frac{f(x)}{g(x)}$

$\therefore \quad \frac{f(x)}{g(x)} = \frac{t^2+2}{t} = t + \frac{2}{t}$

Let $h(t) = t + \frac{2}{t}$

$h'(t) = 1 - \frac{2}{t^2}$

$\therefore$ Local minimum value occurs at $t = \sqrt{2}$

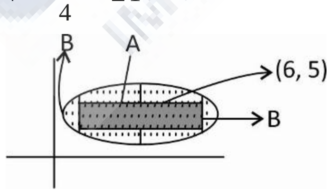
$\therefore$ Local minimum value $= h(\sqrt{2}) = \sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

41.(4) Since Set A is, $|a-5|<1 \quad 4 < a < 6$

and $\quad |b-5|<1 \quad 4 < b < 6$

Now B is

$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \le 1$



It can be seen that all vertices of rectangle lie inside the ellipse, therefore $A \subset B$

42.(3) $\sim(p \vee q) \vee (\sim p \wedge q)$

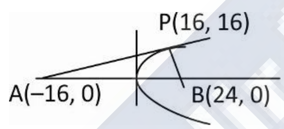
p	q	$\sim(p \vee q)$	$\sim p \wedge q$	$\sim p$
T	F	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	F	T

43.(4) The equation of tangent at P

$$y-16=\frac{1}{2}(x-16) \quad \Rightarrow \quad A \equiv (-16, 0)$$

The normal is $y-16=-2(x-16)$

$B \equiv (24, 0)$



Since $\angle APB = \frac{\pi}{2}$

$\therefore$ AB is the diameter.

Center of the circle $C \equiv (4, 0)$

Slope of $PB = -2 = m_1$

Slope of $CP = \frac{4}{3} = m_2 \Rightarrow \tan\theta = \left|\frac{m_2 - m_1}{1+m_2 m_1}\right| = 2$

44.(1) $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put x = 0

$\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3$

$\boxed{A=-4}$

Put x = 1

$\begin{vmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{vmatrix} = (A+B)(1-A)^2$

$-3(9-4)-2(-6-4)+2(4+6)$

$-15+20+20=(-4+B)25$

$1=(-4+B)$

B = 5

45.(2) Let $\sqrt{x^3-1}=y$

$(x+y)^5+(x-y)^5$

$=\left({}^5C_0x^5+{}^5C_1x^4y+.......+{}^5C_5\,y^5\right)+\left({}^5C_0x^5-5C_1x^4\cdot y+........-5C_5y^5\right)$

$=2\left[5C_0x^5+{}^5C_2x^3y^2+{}^5C_4xy^4\right]=2\left[C_0x^5+5C_2x^3y^2+{}^5C_4xy^4\right]$

$=2\left[x^5+10x^3\left(x^3-1\right)+5x\left(x^3-1\right)^2\right]=2\left[x^5+10x^6-10x^3+5x\left(x^6+1-2x^3\right)\right]$

$= 2\left[x^5 + 10x^6 - 10x^3 + 5x^7 + 5x - 10x^4\right] = 2[1 - 10 + 5 + 5] = 2$

46.(1) $a_1 + a_5 + a_9 + ...a_{49} = 416 \quad \Rightarrow \quad a + 24d = 32 \quad ......(i)$

$a_9 + a_{43} = 66 \quad \Rightarrow \quad a + 25d = 33 \quad ....(ii)$

From (i) and (ii) $d = 1$ and $a = 8$

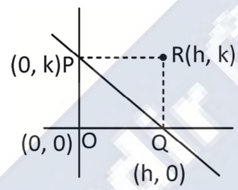
Now, $a_1^2 + a_2^2 + ...a_{17}^2 = 140m$

$\Rightarrow \sum_{r=1}^{17}\left(8 + (r-1)\right)^2 = 140m \quad \Rightarrow \quad \sum_{r=1}^{17}(7 + r)^2 = 140m \quad \Rightarrow \quad 4760 = 140m \quad \Rightarrow \quad m = 34$

47.(1) Let, $R \equiv (h, k)$

$\therefore \quad P \equiv (0, k)$

$Q \equiv (h, 0)$



$\therefore$ Equation of line would be,

$\frac{x}{h} + \frac{y}{k} = 1 \quad ...(i)$

$\Rightarrow \quad \frac{2}{h} + \frac{3}{k} = 1$

$2k + 3h = hk$

Locus of (h, k) is $2y + 3x = xy$

48.(2) Given $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$

$$f(x) + f(-x) = \frac{\sin^2 x}{1+2^x} + \frac{2^x(\sin^2 x)}{1+2^x} = \sin^2 x = \int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \sin^2 x\,dx = \frac{\pi}{4}$$

49.(3) $g(x) = \cos x^2$

$f(x) = \sqrt{x}$

$g(f(x)) = \cos x$

Given, $18x^2 - 9\pi x + \pi^2 = 0 \quad \Rightarrow \quad (6x - \pi)(3x - \pi) = 0$

$\therefore \quad x = \frac{\pi}{6}, \frac{\pi}{3}$

Area = $\int_{\pi/6}^{\pi/3} \cos x\; dx = \frac{\sqrt{3} - 1}{2}$

50.(1) $\lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right)$

$= \lim_{x \to 0^+} x\left(\frac{1}{x} - \left\{\frac{1}{x}\right\} + \frac{2}{x} - \left\{\frac{2}{x}\right\} + ... + \frac{15}{x} - \left\{\frac{15}{x}\right\}\right)$

$= \lim_{x \to 0^+} (1 + 2 + 3 + ... + 15) + \lim_{x \to 0^+} x\left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + ... + \left\{\frac{15}{x}\right\}\right)$

Now $0 \le \{x\} < 1 \;\forall\; x \in R \; = \; 120$

51.(1) Variance = $\frac{45}{9} - (1)^2 \quad = 5 - 1 = 4$

$\sigma = \sqrt{\text{Variance}} = 2$

52.(4) $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$

$\int \frac{\tan^2 x \sec^6 x}{\left(\tan^5 x + \tan^2 x + \tan^3 x + 1\right)^2} dx$

Put $\underline{\tan x = t}$ $\Rightarrow$ $\sec^2 x = \frac{dt}{dx}$

$\int \frac{t^2 \quad (1+t^2)^2}{(t^3+1)^2 (t^2+1)^2} dt$

$t^3 + 1 = y$

$3t^2 = \frac{dy}{dt}$

$\frac{1}{3}\int \frac{dy}{y^2} = -\frac{1}{3(y)} + C = -\frac{1}{3(\tan^3 x + 1)} + C$

53.(3) Doubtful points for differentiability are 0 and π

At $x = 0$

$f'(0^+) = \lim_{h \to o^+} \frac{|h - \pi| \times (e^{|h|} - 1) \times \sin|h| - 0}{h}$

$= \lim_{h \to o^+} \frac{(\pi - h) \times (e^h - 1) \times \sin h}{h}$

$\because \lim_{h \to 0^+} \frac{\sinh}{h} = 1$ and $\lim_{h \to o^+} e^h - 1 = 0$

$\therefore \quad f'(0^+) = \pi \times 0 \times 1 = 0$

$f'(0^-) = \lim_{h \to o^+} \frac{|-h - \pi| \times (e^{|-h|} - 1) \times \sin|-h| - 0}{-h}$

$= \lim_{h \to 0^+} \frac{(\pi + h) \times (e^h - 1) \times \sin h}{-h}$

$\because \lim_{h \to 0^+} \frac{\sinh}{h} = 1$ and $\lim_{h \to o^+} e^h - 1 = 0$

$\therefore \quad f'(0^-) = (-\pi) \times 0 \times 1 = 0$

$\therefore \quad f'(0^+) = f'(0^-) = 0$

Similarly $f'(\pi^+) = f'(\pi^-) = 0$

Hence $f(x)$ is differentiable $\forall\ x \in R$

54.(1) $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x \quad \Rightarrow \quad d\left(y \sin x\right) = 4x dx$

Integrating both sides we get: $y \sin x = 2x^2 + c$

Also, $y\left(\frac{\pi}{2}\right) = 0 \quad \Rightarrow \quad c = -\frac{\pi^2}{2}$

$\Rightarrow \quad y \sin x = 2x^2 - \frac{\pi^2}{2} \quad \Rightarrow \quad y\left(\frac{\pi}{6}\right) = -\frac{8\pi^2}{9}$

55.(3) $\vec{u}\cdot(\vec{a}\times\vec{b})=0;\quad \vec{u}\cdot\vec{a}=0$ and $\vec{u}\cdot\vec{b}=24.$

Let $\vec{b}=(\vec{b}\cdot\hat{a})\hat{a}+(\vec{b}\cdot\hat{u})\hat{u}$

$$|\vec{b}|^2=(\vec{b}\cdot\hat{a})^2+(\vec{b}\cdot\hat{u})^2$$

$$|\vec{b}|^2=(\vec{b}\cdot\hat{a})^2+\frac{(\vec{b}\cdot\hat{u})^2}{|\hat{u}|^2}$$

$$2=\frac{2}{7}+\frac{(24)^2}{|\hat{u}|^2}\quad\Rightarrow\quad |\vec{u}|^2=336$$

56.(2) $\frac{x-5}{1}=\frac{y+1}{1}=\frac{z-4}{1}=\lambda$

$P(\lambda+5,\ \lambda-1,\ \lambda+4)$

P is foot of perpendicular from A to plane $3\lambda+8=7$

$$\lambda=-\frac{1}{3}$$

$$P\left(\frac{14}{3},\frac{-4}{3},\frac{11}{3}\right)$$

$$\frac{x-4}{1}=\frac{y+1}{1}=\frac{z-3}{1}$$

$$Q(\lambda+4,\lambda-1,\lambda+3)$$

Q is foot of perpendicular from B to plane

$$3\lambda+6=7$$

$$\lambda=\frac{1}{3}$$

$$Q\left(\frac{13}{3},\frac{-2}{3},\frac{10}{3}\right)$$

$$\therefore\quad PQ=\frac{\sqrt{1+4+1}}{3}=\frac{\sqrt{6}}{3}=\sqrt{\frac{2}{3}}$$

57.(3) $\frac{h}{x}=\frac{1}{\sqrt{3}}$

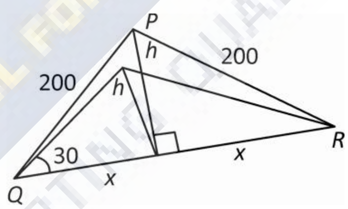
$x=\sqrt{3}h$

$200=3h^2+h^2$

$4h^2=(200)^2$

$4h^2=40000$

$h=100$



58.(3) ${}^6C_4\cdot{}^3C_1\times1\times4!$

$\frac{6\times5}{2}\cdot3\times24\equiv45\times24=\boxed{1080}$

59.(4) $A=1^2+2.2^2+3^2+2.4^2+........+A^2+2.20^2$

$$=\left(1^2+2^2+3^2+4^2.........+20^2\right)+\left(2^2+4^2+.......+20^2\right)$$

$$=\frac{20\times21\times41}{6}+4\times\frac{10\times11\times21}{6}=2870+1540=4410=2870+1540=4410$$

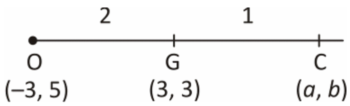
$$B = \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6} = 540 \times 41 + 41 \times 280 = 41 \times 820 = 33620$$

$33620 - 8820 = 100\lambda$

$100\lambda = 24800$

$\lambda = 248$

60.(1)



$\frac{2a-3}{3} = 3 \quad \Rightarrow \quad 2a = 12 \Rightarrow a = 6$

$\frac{2b+5}{3} = 3 \quad \Rightarrow \quad 2b = 4 \Rightarrow b = 2$

$AC = \sqrt{(6+3)^2 + 3^2}$

Diameter $= AC = \sqrt{81+9} = \sqrt{90}$

Radius $= \frac{3\sqrt{10}}{2} = \frac{3 \times \sqrt{10}}{\sqrt{2} \times \sqrt{2}} \qquad = 3\sqrt{\frac{5}{2}}$