JEE(MAIN) – 2018 TEST PAPER WITH SOLUTION (HELD ON SUNDAY 08th APRIL, 2018)

PART C – PHYSICS

61. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

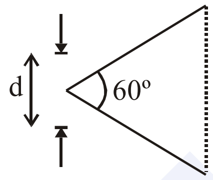
(1) 50 μm (2) 75 μm

(3) 100 μm (4) 25 μm

Ans. **(4)**

Sol. In diffraction

$d \sin 30^\circ = \lambda$



$$\lambda = \frac{d}{2}$$

Young's fringe width

[d' – separation between two slits]

$$\beta = \frac{\lambda \times D}{d'}$$

$$10^{-2} = \frac{d}{2} \times \frac{50 \times 10^{-2}}{d'}$$

$$10^{-2} = \frac{10^{-6} \times 50 \times 10^{-2}}{2 \times d'}$$

$$\boxed{d' = 25\mu m}$$

62. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$

(3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$

Ans. **(4)**

Sol. $\lambda_n = \frac{h}{mu} = \frac{h}{\sqrt{2mk_n}}$

$$\Rightarrow k_n = \frac{h^2}{2m\lambda_n^2} \ ; \ k_g = \frac{h^2}{2m\lambda_g^2}$$

$$\Rightarrow k_g - k_n = \frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]$$

$E_n = -k_n$

for emitted photon

$$\frac{hc}{\Lambda_n} = E_n - E_g = K_g - K_n$$

$$\frac{1}{\Lambda_n} = \frac{K_g - K_n}{hc}$$

$$\Lambda_n = \frac{hc}{K_g - K_n} \Rightarrow \Lambda_n = \frac{hc}{\frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]}$$

$$\Lambda_n = \frac{2mc}{h\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2\lambda_n^2}\right)}$$

$$\Lambda_n = \frac{2mc\lambda_g^2\lambda_n^2}{h\left(\lambda_n^2 - \lambda_g^2\right)}$$

as $\lambda_n \propto n$

$\lambda_n >> \lambda_g$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$

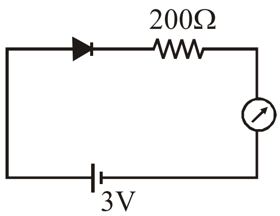
$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2 + \text{higher powers of } \frac{\lambda_g}{\lambda_n}\right]$$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

where $A = \frac{2mc\lambda_g^2}{h}$

& $B = \frac{2mc\lambda_g^4}{h}$

63. The reading of the ammeter for a silicon diode in the given circuit is :-

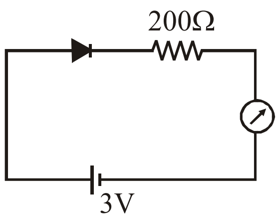


(1) 15 mA (2) 11.5 mA

(3) 13.5 mA (4) 0

Ans. **(2)**

Sol.



Silicon diode is in forward bias.

Hence across diode potential barrier

$\Delta V = 0.7$ volts

$$I = \frac{V - \Delta V}{R} = \frac{3 - 0.7}{200}$$

$$= \frac{2.3}{200} = 11.5 \text{ mA}$$

64. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is :-

(1) 3.5 % (2) 4.5 %

(3) 6 % (4) 2.5 %

Ans. **(2)**

Sol. Density = $\frac{\text{Mass}}{\text{Volume}}$

$$\frac{1\Delta d}{d} = \frac{1\Delta M}{M} + \frac{3\Delta L}{L}$$

= 1.5 + 3(1)

= 4.5 %

65. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:-

(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$

(3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

Ans. **(1)**

Sol. Radius of circular path in magnetic field is given by $R = \frac{\sqrt{2Km}}{qB}$

where K = kinetic energy of particle

m = mass of particle

q = charge on particle

B = magnetic field intensity

R = radius of path

For electron

$$r_e = \frac{\sqrt{2Km_e}}{eB} \quad ...(i)$$

For proton

$$r_p = \frac{\sqrt{2Km_p}}{eB} \quad ...(ii)$$

For α particle

$$r_\alpha = \frac{\sqrt{2Km_\alpha}}{q_\alpha B} = \frac{\sqrt{2K\,4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \quad ...(iii)$$

as $m_e < m_p$ so $r_e < r_p = r_\alpha$

66. Three concentric metal shells A, B and C of respective radii a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is :-

(1) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

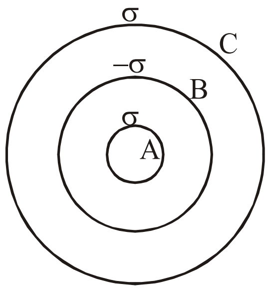
(2) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$

(3) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

(4) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$

Ans. (1)

Sol.



$V_{outside} = \frac{KQ}{r}$

where r is distance of point from the centre of shell $V_{inside} = \frac{KQ}{R}$

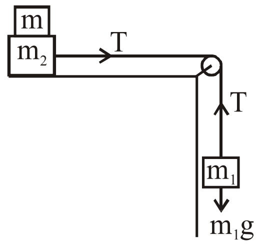
where 'R' is radius of the shell

$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$$

67. Two masses m_1 = 5kg and m_2 = 10kg, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is :-

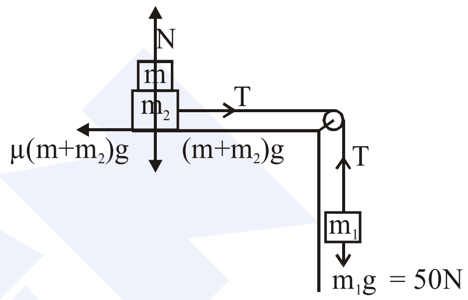


(1) 27.3 kg (2) 43.3 kg

(3) 10.3 kg (4) 18.3 kg

Ans. (1)

Sol.



$50 - T = 5 \times a$

$T - 0.15\ (m + 10)\ g = (10 + m)a$

a = 0 for rest

$50 = 0.15\ (m + 10)\ 10$

$5 = \frac{3}{20}(m + 10)$

$\frac{100}{3} = m + 10$

m = 23.3 kg

68. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is :-

(1) $\frac{k}{2a^2}$ (2) Zero

(3) $-\frac{3}{2}\frac{k}{a^2}$ (4) $-\frac{k}{4a^2}$

Ans. (2)

Sol. $F = -\frac{\partial u}{\partial r} = \frac{K}{r^3}$

Since it is performing circular motion

$$F = \frac{mv^2}{r} = \frac{K}{r^3}$$

$$mv^2 = \frac{K}{r^2}$$

$$\Rightarrow \text{K.E.} = \frac{1}{2}mv^2 = \frac{K}{2r^2}$$

Total energy = P.E. + K.E.

$$= -\frac{K}{2r^2} + \frac{K}{2r^2} = \text{Zero}$$

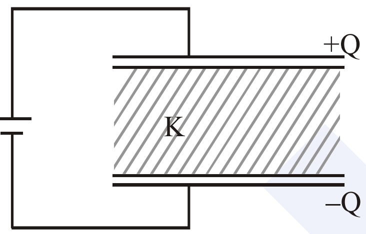
69. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :-

(1) 0.3 n C (2) 2.4 n C

(3) 0.9 n C (4) 1.2 n C

Ans. **(4)**

Sol.



$Q = (kC)\ V$

$$= \left(\frac{5}{3} \times 90\text{pF}\right)(20\text{V})$$

= 3000 pC

= 3nC

induced charges on dielectric

$$Q_{ind} = Q\left(1 - \frac{1}{K}\right) = 3\text{nC}\left(1 - \frac{3}{5}\right) = 1.2\text{nC}$$

70. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(1) 7.1 N/m (2) 2.2 N/m

(3) 5.5 N/m (4) 6.4 N/m

Ans. **(1)**

Sol. Time period of SHM is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\text{frequency} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

where m = mass of one atom

$$= \frac{108}{(6.02 \times 10^{23})} \times 10^{-3}\text{kg}$$

$$\frac{1}{2\pi}\sqrt{\frac{k}{108 \times 10^{-3}} \times 6.02 \times 10^{23}} = 10^{12}$$

On solving K = 7.1 N/m

71. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively :

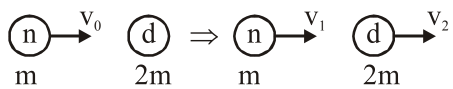
(1) (·28, ·89) (2) (0, 0)

(3) (0, 1) (4) (·89, ·28)

Ans. **(4)**

Sol. Let initial speed of neutron is v_0 and kinetic energy is K.

1st collision :



by momentum conservation

$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0$

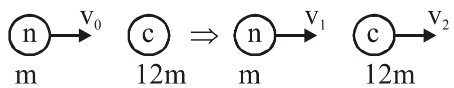
by e = 1 $v_2 - v_1 = v_0$

$$\Rightarrow \quad v_2 = \frac{2v_0}{3}\ ;\ v_1 = -\frac{v_0}{3}$$

$$\text{fractional loss} = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$$

$$\Rightarrow P_d = \frac{8}{9} \approx .89$$

2nd collision :



by momentum conservation

$$mv_0 = mv_1 + 12mv_2$$

$$\Rightarrow v_1 + 12v_2 = v_0$$

by e = 1 $\quad v_2 - v_1 = v_0$

$$v_2 = \frac{2v_0}{13} \quad ; \quad v_1 = \frac{-11v_0}{13}$$

Now fraction loss of energy

$$P_c = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} = \frac{48}{169} \approx 0.28$$

72. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is :

(1) $\sqrt{3}$ (2) $\sqrt{2}$

(3) $\frac{1}{\sqrt{2}}$ (4) 2

Ans. **(2)**

Sol. Dipole moment of circular loop is m

$m_1 = I.A = I.\pi R^2$ {R = radius of the loop}

$$B_1 = \frac{\mu_0 I}{2R}$$

moment becomes double

$\Rightarrow$ R becomes $\sqrt{2}R$ (keeping current constant)

$m_2 = I.\pi(\sqrt{2}R)^2 = 2.I\pi R^2 = 2m_1$

$$B_2 = \frac{\mu_0 I}{2(\sqrt{2}R)} = \frac{B_1}{\sqrt{2}}$$

$$\frac{B_1}{B_2} = \sqrt{2}$$

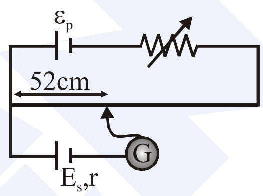
73. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(1) 1.5 Ω (2) 2 Ω

(3) 2.5 Ω (4) 1 Ω

Ans. **(1)**

Sol. without shunting condition :

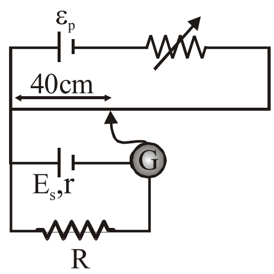


$E_s = 52 \times x$ ...(1)

when balanced

where, x = potential gradient of wire.

with shunting condition



On balancing

$$E_s - \frac{E_s}{(r+R)}r = 40 \times x \quad ....(2)$$

On solving :

$$\frac{(1)}{(2)} \Rightarrow \frac{1}{1-\frac{r}{r+R}} = \frac{52}{40} \quad \therefore r = 1.5\ \Omega$$

74. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz ?

(1) 2×10^4 (2) 2×10^5

(3) 2×10^6 (4) 2×10^3

Ans. (2)

Sol. Since the carrier frequency is distributed as band width frequency, so

10% of 10 GHz = n × 5 kHz

where n = no of channels

$$\frac{10}{100} \times 10 \times 10^9 = n \times 5 \times 10^3$$

$n = 2 \times 10^5$ telephonic channels

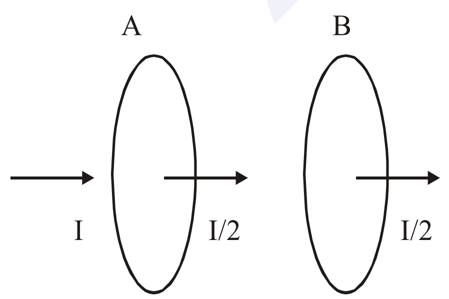
75. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{1}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is :

(1) 30° (2) 45°

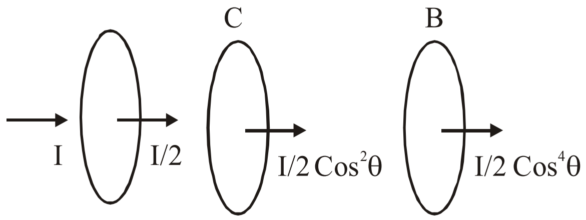
(3) 60° (4) 0°

Ans. (2)

Sol. Axis of transmission of A & B are parallel.



Now,



$$\frac{I}{2}\cos^4\theta = \frac{I}{8} \Rightarrow \cos^4\theta = \frac{1}{4}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$\theta = 45°$

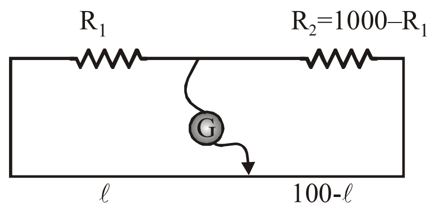
76. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ?

(1) 505 kΩ (2) 550 kΩ

(3) 910 kΩ (4) 990 kΩ

Ans. (2)

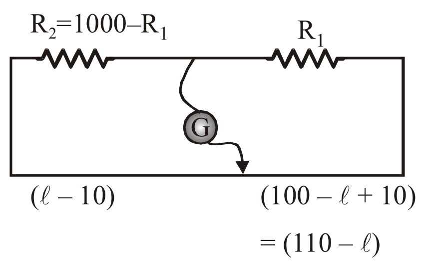
Sol. $R_1 + R_2 = 1000 \Rightarrow R_2 = 1000 - R_1$



On balancing condition

$R_1(100 - \ell) = (1000 - R_1)\ell$ ...(1)

On Inter changing resistance



On balancing condition

$(1000 - R_1)(110 - \ell) = R_1(\ell - 10)$

or $R_1(\ell - 10) = (1000 - R_1)(110 - \ell)$ ....(2)

(1) ÷ (2)

$$\frac{100-\ell}{\ell-10}=\frac{\ell}{110-\ell}$$

$\Rightarrow (100-\ell)(110-\ell) = \ell(\ell-10)$

$\Rightarrow 11000 - 100\ell - 110\ell + \ell^2 = \ell^2 - 10\ell$

$\Rightarrow 11000 = 200\ell$

$\ell = 55$

Put in eq(1)

$R_1(100 - 55) = (1000-R_1)55$

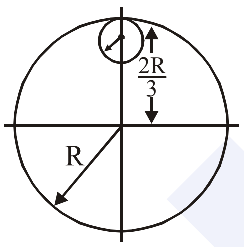
$R_1(45) = (1000-R_1)55$

$R_1(9) = (1000 - R_1)11$

$20\,R_1 = 11000$

$R_1 = 550$

77. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :



(1) $\frac{40}{9}MR^2$ (2) $10\ MR^2$

(3) $\frac{37}{9}MR^2$ (4) $4\ MR^2$

Ans. (4)

Sol. MOI of removed part about axis passing through COM & ⊥ to plane of disc

$= I_{cm} + md^2$

$$=\frac{(m)\,(R/3)^2}{2}+m\left[\frac{4R^2}{9}\right]=\frac{mR^2}{2}$$

so MOI of remaining portion

= [MOI of whole disc – MOI of removed part]

$$=(9m)\frac{R^2}{2}-\frac{mR^2}{2}=\frac{mR^2}{2}[8]$$

$I_{remaining} = 4mR^2$

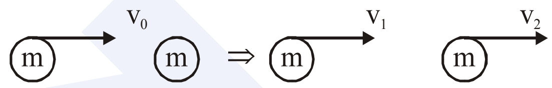
78. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is :

(1) $\sqrt{2}\,v_0$ (2) $\frac{v_0}{2}$

(3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

Ans. (1)

Sol. initial



$$\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2=\frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow v_1^2+v_2^2=\frac{3}{2}v_0^2 \qquad ...(1)$$

from momentum conservation

$mv_0 = m(v_1 + v_2)$ ...(2)

$(v_1 + v_2)^2 = v_0^2$

$\Rightarrow v_1^2+v_2^2+2v_1v_2=v_0^2$

$$2v_1v_2=-\frac{v_0^2}{2}$$

$$(v_1-v_2)^2 = v_1^2+v_2^2-2v_1v_2=\frac{3}{2}v_0^2+\frac{v_0^2}{2}$$

$v_1 - v_2 = \sqrt{2}v_0$

79. An EM wave from air enters a medium. The electric fields are

$\vec{E}_1=E_{01}\hat{x}\cos\left[2\pi\nu\left(\frac{z}{c}-t\right)\right]$ in air and

$\vec{E}_2=E_{02}\hat{x}\cos\left[k(2z-ct)\right]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivity of air and medium respectively, which of the following options is correct ?

(1) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$ (2) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

(3) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$ (4) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$

Ans. (2)

Sol. velocity of EM wave is given by $\frac{1}{\sqrt{\mu \epsilon}}$

velocity in air $= \frac{\omega}{k} = C$

velocity in medium $= \frac{C}{2}$

$$\therefore \frac{\frac{1}{\sqrt{\epsilon_{r_1}}}}{\frac{1}{\epsilon_{r_2}}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \Rightarrow \frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$$

80. For an RLC circuit driven with voltage of amplitude v_m and frequency $w_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by :

(1) $\frac{\omega_0 R}{L}$ (2) $\frac{R}{(\omega_0 C)}$

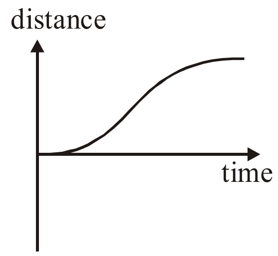
(3) $\frac{CR}{\omega_0}$ (4) $\frac{\omega_0 L}{R}$

Ans. (4)

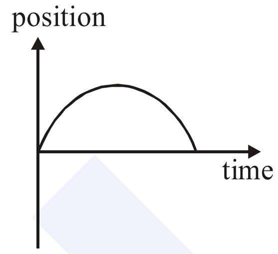
Sol. Quality factor $= \frac{\omega_0 L}{R}$

81. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

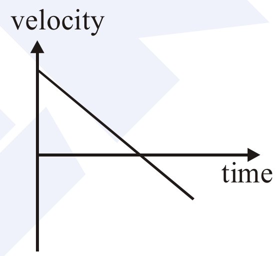
(1)



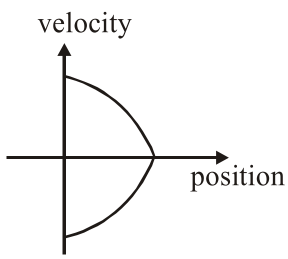
(2)



(3)

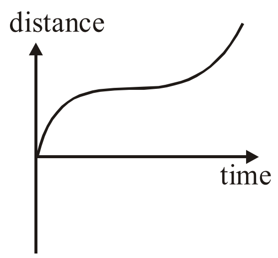


(4)



Ans. (1)

Sol. In this question option (2) and (4) are the corresponding position - time graph and velocity –position graph of option (3) and its distance – time graph is given as



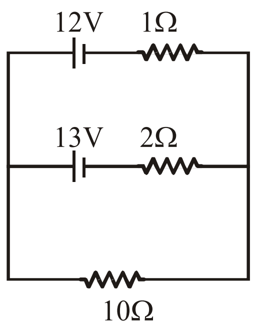
Hence incorrect graph is option (1)

82. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω respectively. The voltage across the load lies between.

(1) 11.5 V and 11.6 V (2) 11.4 V and 11.5 V

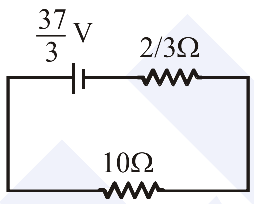
(3) 11.7 V and 11.8 V (4) 11.6 V and 11.7 V

Ans. (1)



Sol. $\dfrac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}}=\dfrac{37}{3}$ volt

$r_{eq}=\dfrac{2\times1}{2+1}=\dfrac{2}{3}\Omega$



Now its equivalent circuit is :

$i=\dfrac{37/3}{10+\frac{2}{3}}=\dfrac{37}{32}$

$\therefore V_{10\Omega}=i\times10=\dfrac{37}{32}\times10=\dfrac{370}{32}=11.56$ volt

Hence (1)

83. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then,

(1) $T\propto R^{\frac{n}{2}+1}$ (2) $T\propto R^{(n+1)/2}$

(3) $T\propto R^{n/2}$ (4) $T\propto R^{3/2}$ for any n

Ans. (2)

Sol. $m\omega^2R$ = Force $\propto \dfrac{1}{R^n}$

$\Rightarrow \omega^2\propto\dfrac{1}{R^{n+1}}$

$\Rightarrow \omega\propto\dfrac{1}{R^{\frac{n+1}{2}}}$

time period $T=\dfrac{2\pi}{\omega}$

$\propto R^{\frac{n+1}{2}}$

84. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is :

(1) 16 ν_L (2) $\nu_L/16$

(3) $\nu_L/25$ (4) 25 ν_L

Ans. (3)

Sol. $h\nu=E_0\left[\dfrac{1}{n_1^2}-\dfrac{1}{n_2^2}\right]$

for Lyman series for series limit $n_2=\infty$, $n_1=1$

$h\nu_L=E_0[1]$ ...(1)

for p-fund series

for series limit $n_2=\infty$, $n_1=5$

$h\nu_p=E_0\left[\dfrac{1}{25}\right]$ ...(2)

By dividing equation (1) and (2)

$\dfrac{\nu_L}{\nu_p}=\dfrac{25}{1}\Rightarrow \nu_p=\nu_L/25$

85. In an a. c. circuit, the instantaneous e.m.f. and current are given by

$e=100\sin 30t$

$i=20\sin\left(30t-\dfrac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively.

(1) $\frac{1000}{\sqrt{2}}$, 10 (2) $\frac{50}{\sqrt{2}}$, 0

(3) 50, 0 (4) 50, 10

Ans. (1)

Sol. $P_{avg} = V_{rms} I_{rms} \cos\theta$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta$$

$$= \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45^\circ$$

$$= \frac{1000}{\sqrt{2}} \text{ watt}$$

wattless current = $I_{rms} \sin\theta$

$$= \frac{I_0}{\sqrt{2}}\sin\theta$$

$$= \frac{20}{\sqrt{2}} \sin 45^\circ$$

= 10 amp.

86. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(1) (a) 195 K (b) –2.7 kJ

(2) (a) 189 K (b) –2.7 kJ

(3) (a) 195 K (b) 2.7 kJ

(4) (a) 189 K (b) 2.7 kJ

Ans. (2)

Sol. In an adiabatic process

PV^{γ} = constant

and, PV = nRT, gives

$$\Rightarrow V^{\gamma-1} \propto \frac{1}{T}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

monoatomic gas : $\gamma = \frac{5}{3}$

$$\Rightarrow T_2 = (300 \text{ K})\left(\frac{V}{2V}\right)^{\frac{5}{3}-1}$$

= 189 K (final temperature)

change in internal energy

$$\Delta U = n\frac{f}{2}R\ \Delta T$$

$$= 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111)$$

= –2.7 kJ

87. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is :

(1) $\frac{Ka}{3mg}$ (2) $\frac{mg}{3Ka}$

(3) $\frac{mg}{Ka}$ (4) $\frac{Ka}{mg}$

Ans. (2)

Sol. [Bulk Modulus = $\frac{\text{volumetric stress}}{\text{volumetric strain}}$]

$$K = \frac{mg}{a\left(\frac{dV}{V}\right)}$$

$$\frac{dV}{V} = \frac{mg}{Ka} \quad \text{...(i)}$$

volume of sphere → $V = \frac{4}{3}\pi R^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ ...(ii)

U sing eq. (i) & (2) $\frac{3dr}{r} = \frac{mg}{Ka}$

$\frac{dr}{r} = \frac{mg}{3Ka}$

88. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ?

(1) 2.5 kHz (2) 10 kHz

(3) 7.5 kHz (4) 5 kHz

Ans. (4)

Sol. Velocity of wave = $\sqrt{\frac{Y}{\rho}}$

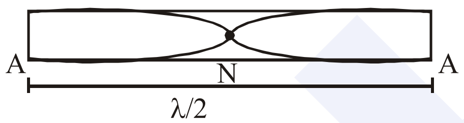
$= \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$

$= \sqrt{3.433 \times 10^7}$

$= 10^3 \times \sqrt{34.33}$

$v_\omega = 5.85 \times 10^3$ m/sec.

Since rod is clamped at middle fundamental wave shape is as follow



$\frac{\lambda}{2} = L$

$\lambda = 2L$

L = 60 cm = 0.6 m (given)

$\lambda = 1.2$ m

$v = f\lambda$

$f = \frac{v}{\lambda} = \frac{5.85 \times 10^3}{1.2}$

$= 4.88 \times 10^3 \text{ Hz} \simeq 5$ KHz

89. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly :

(1) 4.70×10^3 N/m^2

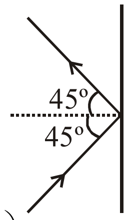
(2) 2.35×10^2 N/m^2

(3) 4.70×10^2 N/m^2

(4) 2.35×10^3 N/m^2

Ans. (4)

Sol. $F = \frac{dp}{dt} = 2n\,mv\cos 45^\circ$

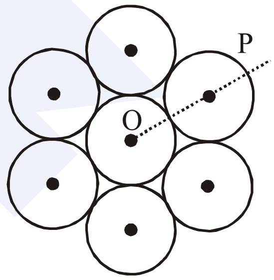


Pressure $= \frac{F}{A} = \frac{2n\,mv\cos 45^\circ}{\text{Area}}$

$= \frac{2 \times 10^{23} \times 3.3 \times 10^{-27} \times 10^3 \times \left(\frac{1}{\sqrt{2}}\right)}{2 \times 10^{-4}}$

$= 2.35 \times 10^3$ N/m^2

90. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is :

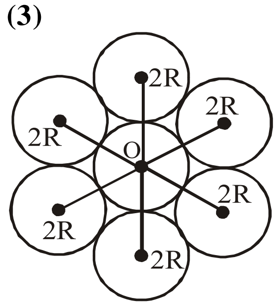


(1) $\frac{55}{2}$ MR^2 (2) $\frac{73}{2}$ MR^2

(3) $\frac{181}{2}$ MR^2 (4) $\frac{19}{2}$ MR^2

Ans. (3)

Sol.



$I_0 = I_{cm} + md^2$

$= \frac{7MR^2}{2} + 6(M \times (2R)^2) = \frac{55MR^2}{2}$

$I_p = I_0 + md^2$

$= \frac{55MR^2}{2} + 7M(3R)^2 = \frac{181}{2}MR^2$